

Sustav linearnih jednadžbi

1) Provjeri je li uređeni par (2, -1) rješenje sustava:

$$\begin{aligned} 3x + y &= 5 \\ \underline{2x - 5y} &= \underline{-9} \end{aligned}$$

Rješenje:

(2, -1) → uvrštavamo u jednadžbu $x = 2$, $y = -1$

Prva jednadžba:

$$\begin{aligned} 3 \cdot 2 + (-1) &= 5 \\ 6 - 1 &= 5 \\ 5 &= 5 \quad \checkmark \end{aligned}$$

Druga jednadžba:

$$\begin{aligned} 2 \cdot 2 - 5 \cdot (-1) &= -9 \\ 4 + 5 &= -9 \\ 9 &= -9 \quad \text{X} \end{aligned}$$

Uređeni par (2, -1) **nije** rješenje sustava.

2) Provjeri je li uređeni par (3, -2) rješenje sustava:

$$\begin{aligned} 5x + 3y &= 9 \\ \underline{-3x + 4y + 9} &= \underline{-8} \end{aligned}$$

Rješenje:

U početne jednadžbe uvrštavamo $x = 3$, $y = -2$:

Prva jednadžba:

$$\begin{aligned} 5 \cdot 3 + 3 \cdot (-2) &= 9 \\ 15 - 6 &= 9 \\ 9 &= 9 \quad \checkmark \end{aligned}$$

Druga jednadžba:

$$\begin{aligned} -3 \cdot 3 + 4 \cdot (-2) + 9 &= -8 \\ -9 - 8 + 9 &= -8 \\ -8 &= -8 \quad \checkmark \end{aligned}$$

Uređeni par (3, -2) **je** rješenje sustava.

3) Metodom supstitucije riješi sustav:

$$\begin{aligned} \text{a) } x - 4y &= -17 \\ \underline{2x + 3y} &= \underline{-1} \end{aligned}$$

$$\begin{aligned} \text{b) } 5x - 2y + 13 &= 0 \\ \underline{x - 2y + 9} &= \underline{0} \end{aligned}$$

$$\begin{aligned} \text{c) } 2x + 3y &= -2 \\ \underline{3x - y + 14} &= \underline{0} \end{aligned}$$

Rješenje:

$$\text{a) } \begin{aligned} x - 4y &= -17 \\ \underline{2x + 3y} &= \underline{-1} \end{aligned} \quad \rightarrow \quad \boxed{x = -17 + 4y} \quad \dots \text{ supstitucija (uvrštavam u 2. jednadžbu)}$$

$$\begin{aligned} 2(-17 + 4y) + 3y &= -1 && \dots \text{ broj množi zagradu} \\ -34 + 8y + 3y &= -1 && \dots \text{ nepoznanice na lijevu stranu, brojevi na desnu} \\ 11y &= 33 \quad :11 \\ y &= 3 \end{aligned}$$

Vraćamo supstituciju (uvrštavamo $y = 3$ u supstituciju ili bilo koju od zadanih jednadžbi):

$$\begin{aligned} x &= -17 + 4y \\ x &= -17 + 4 \cdot 3 \\ x &= -17 + 12 \\ x &= -5 \end{aligned}$$

Rješenje je uređeni par (-5, 3).

$$\text{b) } \begin{array}{l} 5x - 2y + 13 = 0 \\ \underline{x - 2y + 9 = 0} \end{array}$$

Prvo svedemo sustav na **standardni oblik**:

$$\begin{array}{l} 5x - 2y = -13 \\ \underline{x - 2y = -9} \end{array}$$

→

$$x = -9 + 2y$$

(izrazili iz druge jednačbe, pa uvrštavamo u prvu)

$$\begin{array}{l} 5 \cdot (-9 + 2y) - 2y = -13 \\ -45 + 10y - 2y = -13 \\ 8y = 32 / :8 \\ y = 4 \end{array}$$

Vraćamo supstituciju:

$$\begin{array}{l} x = -9 + 2y \\ x = -9 + 2 \cdot 4 \\ x = -1 \end{array}$$

Rješenje je uređeni par (-1, 4).

$$\text{c) } \begin{array}{l} 2x + 3y = -2 \\ \underline{3x - y + 14 = 0} \end{array}$$

Prvo svedemo sustav na **standardni oblik**:

$$\begin{array}{l} 2x + 3y = -2 \\ \underline{3x - y = -14} \end{array}$$

→

$$-y = -14 - 3x / \cdot (-1)$$

$$y = 14 + 3x$$

(uvrštavamo u prvu jednačbu)

$$\begin{array}{l} 2x + 3 \cdot (14 + 3x) = -2 \\ 2x + 42 + 9x = -2 \\ 11x = -44 / :11 \\ x = -4 \end{array}$$

Vraćamo supstituciju:

$$\begin{array}{l} y = 14 + 3x \\ y = 14 + 3 \cdot (-4) \\ y = 14 - 12 \\ y = 2 \end{array}$$

Rješenje je uređeni par (-4, 2).

4) Metodom supstitucije riješi sustav:

$$\text{a) } \begin{array}{l} 2x - 3y = -1 \\ \underline{-4x + 2y = 6} / :2 \end{array}$$

$$2x - 3y = -1$$

$$\underline{-2x + y = 3} \rightarrow y = 3 + 2x$$

$$\begin{array}{l} 2x - 3(3 + 2x) = -1 \\ 2x - 9 - 6x = -1 \\ -4x = 8 / :(-4) \\ x = -2 \end{array}$$

$$y = 3 + 2 \cdot (-2)$$

$$y = 3 - 4$$

$$y = -1$$

Rj: (-2, -1)

$$\text{b) } \begin{array}{l} 3x + 4y = 1 \rightarrow 3x = 1 - 4y / :3 \\ \underline{5x + 2y = 11} \end{array}$$

$$x = \frac{1}{3} - \frac{4}{3}y$$

$$5\left(\frac{1}{3} - \frac{4}{3}y\right) + 2y = 11$$

$$\frac{5}{3} - \frac{20}{3}y + 2y = 11 / \cdot 3$$

$$5 - 20y + 6y = 33$$

$$-14y = 28 / :(-14)$$

$$y = -2$$

$$x = \frac{1}{3} - \frac{4}{3} \cdot (-2)$$

$$x = \frac{1}{3} + \frac{8}{3}$$

$$x = \frac{9}{3}$$

$$x = 3$$

Rj: (3, -2)

c) $8a - 7b = 16$
 $5a - 7b = 10$ → $5a = 10 + 7b / : 5$
 $a = 2 + \frac{7}{5}b$

$8 \cdot \left(2 + \frac{7}{5}b\right) - 7b = 16$
 $16 + \frac{56}{5}b - 7b = 16 / \cdot 5$
 $80 + 56b - 35b = 80$
 $b = 0$

$a = 2 + \frac{7}{5} \cdot 0$
 $a = 2$

Rj: (2, 0)

5) Metodom suprotnih koeficijenata riješi sustav:

a) $x + 2y = 10 / \cdot 2$
 $5x - 4y = 8$

$2x + 4y = 20$
 $5x - 4y = 8$
 $7x = 28 / : 7$
 $x = 4$

$4 + 2y = 10$
 $2y = 6 / : 2$
 $y = 3$

(4, 3)

b) $5x - 2y + 13 = 0$
 $3x + y - 1 = 0$

$5x - 2y = -13$
 $3x + y = 1 / \cdot 2$
 $5x - 2y = -13$
 $6x + 2y = 2$
 $11x = -11 / : (11)$
 $x = -1$

$3 \cdot (-1) + y = 1$
 $-3 + y = 1$
 $y = 4$

(-1, 4)

c) $7x + 4y = 31 / \cdot 2$
 $-2x + 5y = -15 / \cdot 7$

$14x + 8y = 62$
 $-14x + 35y = -105$
 $43y = -43 / : 43$
 $y = -1$

$-2x + 5 \cdot (-1) = -15$
 $-2x - 5 = -15$
 $-2x = -10 / : (-2)$
 $x = 5$

(5, -1)

6) Riješi sustav:

a) $x + \frac{2}{3}y = \frac{20}{3} / \cdot 3$

$\frac{3}{2}x + \frac{2}{3}y = \frac{14}{3} / \cdot 6$

$3x + 2y = 20 / \cdot (-2)$
 $9x + 4y = 28$
 $-6 - 4y = -40$
 $9x + 4y = 28$
 $3x = -12 / : 3$
 $x = -4$

$3 \cdot (-4) + 2y = 20$
 $-12 + 2y = 20$
 $2y = 32 / : 2$
 $y = 16$

(-4, 16)

b) $\frac{1}{4}x - \frac{3}{5}y = \frac{17}{10} / \cdot 20$

$-\frac{2}{3}x + \frac{1}{2}y = -\frac{7}{3} / \cdot 6$

$5x - 12y = 34$
 $-4x + 3y = -14 / \cdot 4$
 $5x - 12y = 34$
 $-16x + 12y = -56$
 $-11x = -22 / : (-11)$
 $x = 2$

$-4 \cdot 2 + 3y = -14$
 $-8 + 3y = -14$
 $3y = -6 / : 3$
 $y = -2$

(2, -2)

7) Riješi sustav:

$$\begin{array}{l} \text{a) } 3(x-2) - 2(1-3y) = 4 \\ \underline{3(x-3) - (y+1) = -5} \end{array}$$

$$\begin{array}{l} \text{b) } 3(x-2) - 2(2x+y-3) = -9 \\ \underline{5x+2 = 3 - 4 \cdot (3x-y)} \end{array}$$

$$\begin{array}{l} \text{a) } 3x - 6 - 2 + 6y = 4 \\ \underline{3x - 9 - y - 1 = -5} \end{array}$$

$$\begin{array}{l} 3x + 6y = 12 \quad /:3 \\ \underline{3x - y = 5} \end{array}$$

$$\begin{array}{l} x + 2y = 4 \\ \underline{3x - y = 5} \end{array}$$

$$x = 4 - 2y$$

$$\begin{array}{l} 3 \cdot (4 - 2y) - y = 5 \\ 12 - 6y - y = 5 \\ -7y = -7 \quad /: (-7) \\ \mathbf{y = 1} \end{array}$$

$$\begin{array}{l} x = 4 - 2 \cdot 1 \\ x = 4 - 2 \\ \mathbf{x = 2} \end{array}$$

Rješenje je uređeni par (2, 1).

$$\begin{array}{l} \text{b) } 3x - 6 - 4x - 2y + 6 = -9 \\ \underline{5x + 2 = 3 - 12x + 4y} \end{array}$$

$$\begin{array}{l} -x - 2y = -9 \\ \underline{17x - 4y = 1} \end{array}$$

$$\begin{array}{l} -x = -9 + 2y \\ \mathbf{x = 9 - 2y} \end{array}$$

$$\begin{array}{l} 17 \cdot (9 - 2y) - 4y = 1 \\ 153 - 34y - 4y = 1 \\ -38y = -152 \quad /: (-38) \\ \mathbf{y = 4} \end{array}$$

$$\begin{array}{l} x = 9 - 2 \cdot 4 \\ x = 9 - 8 \\ \mathbf{x = 1} \end{array}$$

Rješenje je uređeni par (1, 4).

8) Riješi sustav:

$$\begin{array}{l} \text{a) } \frac{3x-2y}{5} + \frac{5x+3y}{2} = \frac{29}{10} \quad / \cdot 10 \\ \underline{\frac{4x+y}{4} - \frac{3x-5y}{3} = \frac{-4x+5y}{4} \quad / \cdot 12} \end{array}$$

$$\begin{array}{l} 2 \cdot (3x - 2y) + 5 \cdot (5x + 3y) = 29 \\ \underline{3 \cdot (4x + y) - 4 \cdot (3x - 5y) = 3 \cdot (-4x + 5y)} \end{array}$$

$$\begin{array}{l} 6x - 4y + 25x + 15y = 29 \\ \underline{12x + 3y - 12x + 20y = -12x + 15y} \end{array}$$

$$\begin{array}{l} 31x + 11y = 29 \\ \underline{12x + 8y = 0} \quad /:4 \end{array}$$

$$\begin{array}{l} 31x + 11y = 29 \\ \underline{3x + 2y = 0} \quad \rightarrow \quad 2y = -3x \quad /:2 \\ \mathbf{y = -\frac{3}{2}x} \end{array}$$

... svodimo sustav na **STANDARDNI OBLIK**
(nepoznanice na lijevu, brojevi na desnu stranu)

Uvrštavamo supstituciju u 1. jednadžbu:

$$\begin{aligned}31x + 11 \cdot \left(\frac{-3}{2}x\right) &= 29 \\31x - \frac{33}{2}x &= 29 / \cdot 2 \\62x - 33x &= 58 \\29x &= 58 / :29 \\x &= 2\end{aligned}$$

Vraćamo supstituciju:

$$\begin{aligned}y &= \frac{-3}{2}x \\y &= \frac{-3}{2} \cdot 2 \\y &= -3\end{aligned}$$

Rješenje je uređeni par (2, -3).

b) $5(2x + y) - 3(2x - 4y - 1) = 7 - 7(x - 3y + 2)$

$$1 - \frac{y-3}{2} + \frac{2x+5y+10}{6} = \frac{x-2y+6}{4} \quad / \cdot 12$$

$$\begin{aligned}10x + 5y - 6x + 12y + 3 &= 7 - 7x + 21y - 14 \\12 - 6(y-3) + 2(2x+5y+10) &= 3(x-2y+6)\end{aligned}$$

$$\begin{aligned}11x - 4y &= -10 \\12 - 6y + 18 + 4x + 10y + 20 &= 3x - 6y + 18\end{aligned}$$

$$\begin{aligned}11x - 4y &= -10 \\x + 10y &= -32 \quad \rightarrow \quad \boxed{x = -32 - 10y}\end{aligned}$$

$$\begin{aligned}11 \cdot (-32 - 10y) - 4y &= -10 \\-352 - 110y - 4y &= -10 \\-114y &= 342 / : (-114) \\y &= -3\end{aligned}$$

Vraćamo supstituciju:

$$\begin{aligned}x &= -32 - 10 \cdot (-3) \\x &= -32 + 30 \\x &= -2\end{aligned}$$

... svodimo jednadžbe na STANDARDNI OBLIK

... nepoznanice na lijevu, brojevi na desnu stranu
... broj množi zagradu (množi **svaki broj** u zagradi,
pazi na predznak!)

Rješenje je uređeni par (-2, -3).